Even and Odd functions

Do Now

Given \( f(x) = 3x^3 + Ax^2 - Bx + 4 \), \( f(2) = 42 \) and \( f(-2) = -10 \). Find \( A \), \( B \), and the value of \( 3A - 5B \).

Lesson

How do we tell if a function is even?

- To determine algebraically whether a function is even we substitute \(-x\) for \( x \) in the equation. If \( f(-x) = f(x) \) then we have an even function.

1. Determine algebraically whether \( f(x) = x^2 \) is even.
   - \( f(x) = x^2 \)
   - \( f(-x) = (-x)^2 \)
   - \( f(-x) = x^2 \)
     - Do we have the same results we started with? Yes then it’s even

   - What does this graph look like?
   - Graph equation

2. What does an even function look like graphically?
   - It is "symmetric about the y-axis"; in other words, whatever the graph is doing on one side of the \( y \)-axis is mirrored on the other side.

2. Determine Algebraically whether \( f(x) = |x| \) is even.
   - \( f(x) = |x| \)
   - \( f(-x) = |-x| \)
   - \( f(-x) = |x| \)
     - Same results? Yes
3. Determine algebraically whether \( f(x) = -3x^2 + 4 \) is it even.
   - So I'll plug \(-x\) in for \(x\), and simplify:
     \[
     f(-x) = -3(-x)^2 + 4 \\
     = -3(x^2) + 4 \\
     = -3x^2 + 4
     \]

How to do we determine if a function is odd?

- We start with the process from even then find the negation of the original function, which is \(-f(x)\). If \(-f(x)\) is the same as \(f(-x)\), then the function is odd.
  - Does the input of \(-x\) give you \(-y\)

4. Determine algebraically whether \( f(x) = x \) is even or odd.
   - \( f(-x) = -x \)
     - \( -f(x) = -(x) \)
     - \( -f(x) = -x \)

   ODD

5. Determine algebraically whether \( f(x) = x^3 \) even or odd
   - \( f(-x) = (-x)^3 \)
   - \( f(-x) = -x^3 \)
     - \( -f(x) = -(x^3) \)
     - \( -f(x) = -x^3 \)

   Students Do

   Yes, ODD

6. Determine algebraically whether \( f(x) = 2x^3 - 4x \) is even or odd.
   - \( f(-x) = 2(-x)^3 - 4(-x) \)
     \[
     = 2(-x^3) + 4x \\
     = -2x^3 + 4x
     \]
   - \( -f(x) = - (2x^3 - 4x) \)
     - \( -f(x) = -2x^3 + 4x \)
   - Yes, ODD
Can you tell me what an odd function look like?

- It has symmetry with respect to the origin.
  - What does that look like?

Graph $f(x)=x$

Set up xy coordinate chart.

Graph $f(x)=x^3$

Set up xy coordinate chart.

Is $f(x)$ odd?
Neither

7. Determine algebraically whether \( f(x) = 2x^3 - 3x^2 - 4x + 4 \) is even, odd.
   \[
   f(-x) = 2(-x)^3 - 3(-x)^2 - 4(-x) + 4 \\
   = 2(-x^3) - 3(x^2) + 4x + 4 \\
   = -2x^3 - 3x^2 + 4x + 4
   \]
   - Nope, move onto the test for odd.
   \[
   -f(x) = -(2x^3 - 3x^2 - 4x + 4) \\
   = -2x^3 + 3x^2 + 4x - 4
   \]
   - Nope, Neither.

8. Determine whether \( f(x) = x^3 + 2x^2 - x + 3 \) is even, odd, or neither.
   \[
   f(-x) = (-x)^3 + 2(-x)^2 -(-x) + 3 \\
   = -x^3 + 2x^2 + x + 3
   \]
   No so what do we do?
   - Test for oddness.
     \[
     -f(x) = -( x^3 + 2x^2 -x + 3) \\
     = -x^3 - 2x^2 +x - 3
     \]
     - No so is it odd?
     - No so what is it?
     - It is what we will call Neither.

Remember no matter what you are always going to do the test for even. So I never want to see a question blank. There is always a first step to do.
1. Determine algebraically whether \( f(x) = \cos x \)
   - \( f(x) = \cos x \)
   - \( f(-x) = \cos x \)
   - \( f(-x) = \cos (-x) \)
   - \( f(-x) = \cos (0 - x) = \cos 0 \cos x + \sin 0 \sin x \)
   - \( 1 \cos x + 0 \sin x \)
   - \( \cos x + \cos x \).

   But what does the graph look like? This is a good one time to think about the graph. It is easier in this case and the last case to think of the graph than to do it algebraically.
1. State the domain of the function in interval notation.
   a. $f(x) = \sqrt{x^2 - 36}$

   
   b. $f(x) = \frac{15}{\sqrt{x^2 - 4x - 12}}$

   
   c. $f(x) = \frac{6}{x^2 - 16}$

2. State whether the given function is odd, even, or neither. Be sure to justify your response using either a graphic or an algebraic method.

   a. $y = -x^2 + 8$

   
   b. $f(x) = x^3 + x^2$

   
   c. $f(x) = x^4 - x^2 + 7$

   
   d. $xy = -6$
3. How many of the following functions have the property \( f(x) = f(-x) \)?
   a. \( f(x) = x^4 + x^2 \)
   b. \( f(x) = \cos x \)
   c. \( f(x) = 4^x \)
   d. \( f(x) = 17 \)
   e. \( f(x) = x^2 \)
   f. \( y = \sqrt{4-x^2} \)
   g. \( f(x) = x \)

4. If \( f \) and \( g \) are odd and even functions, respectively, such that \( f(a) = b \) and \( g(c) = b \), then \( \frac{f(-a)}{g(-c)} + f(-a) - g(-c) = \)
   a. -1       b. -1-2b      c. -1+2b      d. 1-2b      e. 1+2b

5. If \( w(x) = 5x+10 \) and \( z(x) = x^2 + 6 \), then \( (w^o z^o w^{-1})(0) = \)